Solution of the WIC Model

The chain's profit function for the two brands of infant formula (separate from the other products offered by the chain) is:

$$(6)\ \pi_i(q_{1,i}, q_{2,i}) = \left[\alpha_1 - \beta\left(q_{1,i} + \sum_{j\neq i} q_{1,j}\right) - \gamma\left(q_{2,i} + \sum_{j\neq i} q_{2,j}\right) - c_1\right] q_{1,i}$$
$$+ \left[\alpha_2 - \beta\left(q_{2,i} + \sum_{j\neq i} q_{2,j}\right) - \gamma\left(q_{1,i} + \sum_{j\neq i} q_{1,j}\right) - c_2\right] q_{2,i}$$

For each of the M chains, the first-order conditions are[15]:

$$(7a)\ 0 = \alpha_1 - \beta\left(\sum_{j\neq i} q_{1,j}\right) - \gamma\left(\sum_{j\neq i} q_{2,j}\right) - c_1 - 2\gamma q_{2,i} - 2\beta q_{1,i}$$
$$(7b)\ 0 = \alpha_2 - \beta\left(\sum_{j\neq i} q_{2,j}\right) - \gamma\left(\sum_{j\neq i} q_{1,j}\right) - c_2 - 2\gamma q_{1,i} - 2\beta q_{2,i}$$

In contrast to the Cournot model, for which there is but one best response function (BRF) for each firm, the WIC model's two first-order conditions in (7) each constitute a BRF. Each of the two BRFs exhibit two types of interdependencies. As usual in a Cournot model, (7) exhibits *cross-firm interdependency* for each given brand, i.e., the profit-maximizing level of a brand's output for a chain depends on the given levels of the brand's output chosen by the other chains. In addition, given cross-brand substitution behavior so that $\gamma > 0$, each of the chain's two BRFs exhibit *cross-brand interdependency*, i.e., the profit-maximizing level of output for a chain's brand 1 depends on the firm's own chosen level of output (and its rival's chosen levels of output) for brand 2, and vice versa.

To solve in reduced form for the two optimal quantities of chain *i*, q_{1i}^* and q_{2i}^*, the BRFs must be solved for both cross-firm and cross-brand interactions. As in a Cournot model in which all firms have identical constant marginal costs, each chain's quantity (for a given brand) is identical. Using the result that the output of the $(M - 1)$ rivals for chain i is simply $(M - 1)q_{k,i}$, $k = 1, 2$, (7) becomes, in partially reduced form:

$$(8a)\ q_{1,i}(q_{2,i}) = \frac{\alpha_1 - c_1}{(M+1)\beta} - \frac{\gamma}{\beta} q_{2,i} = \frac{BA_1 + SA_2[B(\theta_1 + h) + S(\theta_2 - h)]Q_W - c_1}{(M+1)\beta} - \frac{S}{B} q_{2,i}$$

$$(8b)\ q_{2,i}(q_{1,i}) = \frac{\alpha_2 - c_2}{(M+1)\beta} - \frac{\gamma}{\beta} q_{1,i} = \frac{BA_2 + SA_1[B(\theta_2 - h) + S(\theta_1 + h)]Q_W - c_2}{(M+1)\beta} - \frac{S}{B} q_{1,i}$$

[15] The second-order conditions $\pi_{11}^i < 0$, $\pi_{22}^i < 0$, and $\pi_{11}^i\pi_{22}^i - (\pi_{12}^i)^2 > 0$ are met at the point at which the first-order conditions are satisfied, given typical assumptions about the magnitude of marginal cost relative to other parameters (to exclude a corner solution of zero output). The solution to (7) constitutes a local profit-maximizing set of output levels for the chain. This local maximum will be the focus of attention, since the infinite price/infinite profits outcome was already ruled out.

Solving (8) simultaneously yields equilibrium quantities for the two brands at the level of a chain:

$$(9a)\ q_{1,i}{*}=\frac{\beta(\alpha_1-c_1)-\gamma(\alpha_2-c_2)}{(\beta^2-\gamma^2)(M+1)}=\frac{[A_1+(\theta_1+h)Q_W]-Bc_1+Sc_2}{(M+1)}$$

$$(9b)\ q_{2,i}{*}=\frac{\beta(\alpha_2-c_2)-\gamma(\alpha_1-c_1)}{(\beta^2-\gamma^2)(M+1)}=\frac{[A_2+(\theta_2-h)Q_W]-Bc_2+Sc_1}{(M+1)}$$

Total production (in the market area) of either brand is the sum across (identical) chains of that brand's output levels, which is simply *M* times greater than the output of any one chain in (9). Equilibrium total production of each brand can be written as:

$$(10a)\ Q_1^*=\frac{M}{(M+1)}[a_1+(\theta_1+h)\delta wvz-bc_1+sc_2]Y$$

$$(10b)\ Q_2^*=\frac{M}{(M+1)}[a_2+(\theta_2-h)\delta wvz-bc_2+sc_1]Y$$

where $Y = H + L$, the number of non-WIC households; $w = W/Y$, the ratio of WIC to non-WIC households who buy formula; and $b = B/Y$, a weighted average of the price-sensitivity parameters (slopes) of the two out-of-pocket groups, which can be written as:

$$(11)\ b=\left(\frac{H}{(H+\mathrm{L})}\right)b_H+\left(\frac{L}{(H+\mathrm{L})}\right)b_L=\omega_H b_H+\omega_L b_L$$

where the shares of *H* and *L* as proportions of all out-of-pocket consumers, ω_H and ω_L, sum to 1.

Using (10) and (4), equilibrium market prices for brands 1 and 2 can be expressed as functions of exogenous terms:

$$(12a)\ P_1^*=\frac{(b\alpha_1+sa_2)}{(M+1)(b^2-s^2)}+\frac{M}{M+1}c_1+\delta\frac{[b(\theta_1+h)+s(\theta_2-h)]vz}{(M+1)(b^2-s^2)}w$$

$$(12b)\ P_2^*=\frac{(b\alpha_2+sa_1)}{(M+1)(b^2-s^2)}+\frac{M}{M+1}c_2+\delta\frac{[b(\theta_2-h)+s(\theta_1+h)]vz}{(M+1)(b^2-s^2)}w$$

The assumption that *b*, the weighted average of the own-price terms, exceeds *s*, the cross-price term, ensures that the denominators in (12) are positive, which means prices for the contract and noncontract brands are positive (given typical assumptions about the relative sizes of demand parameters and marginal cost).